New Method to Find the Axis of a Parabola

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Abstract— The content describes a new way to find the axis of a parabola and a new derivation for the equation of the parabola.

Index Terms— parabola,axis,partial differentiation,

Theorem 1

Consider a parabola $ax^2+2hxy+by^2+2gx+2fy+c=0$, *then*

(i)the slope of the axis is given by $-(\sqrt{a}/\sqrt{b})$

(ii)the equation of the axis of the parabola is given by $ax+hy++((hf+ag)/(a+b))=0$ *which can be simplified as* $\sqrt{a}x+\sqrt{b}y+((f\sqrt{b}+g\sqrt{a})/(a+b))=0$.

Proof.

Consider a parabola of the form

$$ax^2+2hxy+by^2+2gx+2fy+c=0 \quad \textbf{(1)}$$

Then the equation of the parabola can be determined as follows:
Differentiate equation (1) partially with respect to x,
We get $ax+hy+g=0$ **(2)**
Differentiate equation (1) partially with respect to x,
We get $hx+by+f=0$ **(3)**
Since $h^2=a*b$ for a parabola, equations (2) and (3) are Parallel to each other as the slopes are equal.

Consider the point of intersection of the equations (1) and (2)
Put $y=(-ax-g)/h$ in equation (1)….
Then
$ax^2+2x(-ax-g)+b((-ax-g)/h)^2+2gx+2f(-ax-g)/h+c=0$;
consider the x^2 coefficient ,
$(a-2a+b*(a^2)/h^2)=0$ since $h^2=ab$.
Consider the point of intersection of the equations (1) and (3)
Put $y=(-hx-f)/b$ in equation (1)….
Then
$ax^2+2hx(-hx-f)/b+b((-hx-f)/b)^2+2gx+2f(-hx-f)/b+c=0$;
consider the x^2 coefficient
$(a-2(h^2)/b+b*(h^2)/b^2)=0$ since $h^2=ab$.
That means both the lines are parallel and they intersect the parabola at only one point……..
As we cannot draw parallel tangents to a parabola,the lines 2&3 must be parallel to the axis as in the figure(1).
So the SLOPE of the axis of a parabola is (-a/h)or(-h/b).
Since $h^2=ab$,then $(-a/h)=(-a/\sqrt{(ab)})=-(\sqrt{a}/\sqrt{b})$.

Now the second part of the proof starts.

Let us differentiate the equation 1 W.R.T x,
We get $ax+hy+hxD+byD+g+fD=0$; {d refers to (dy/dx)

Then $D= -(ax+hy+g)/(hx+by+f)$ **(4)**
This is the slope of the tangent at any point on the parabola

As the vertex of the parabola the tangent is perpendicular to axis the slope of the tangent at vertex is (h/a) and let the vertex be (p,q).

From (4),

$(h/a)=-(ap+hq+g)/(hp+bq+f)$

Re- arranging the terms gives,

$(a^2+h^2)p+h(a+b)q+hf+ag=0$;

Put $h^2=ab$;

$(a+b)ap+h(a+b)q+hf+ag=0$

It becomes $ap+hq+\{(hf+ag)/(a+b)\}=0$

This equation represents a line which is having slope -(a/h) and passing through vertex (p,q)…

This is notihng but AXIS of the parabola……

So the general equation of the axis is

$ax+hy+\{(hf+ag)/(a+b)\}=0$;
put $h=\sqrt{(ab)}$.
Then $ax+\sqrt{(ab)}y+\{(\sqrt{(ab)}.f+ag)/(a+b)\}=0$
Which implies that
$\sqrt{a}x+\sqrt{b}y+((f\sqrt{b}+g\sqrt{a})/(a+b))=0$.
As the equation of the axis of parabola.

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2 Author`s Bibilography

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